

Minimum fluidization velocities and maximum bed pressure drops for gas–solid tapered fluidized beds

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Abstract

Hydrodynamic characteristics of tapered fluidized beds differ from that of conventional columnar beds by the fact that a velocity gradient exists along the axial direction of the bed. To study the characteristics of tapered beds, several experiments have been carried out with different tapered angles of the bed, with regular as well as irregular particles of different sizes and densities. The tapered angles of the beds have been found to affect the characteristics of the bed. Models based on dimensionless analysis have been proposed to predict the minimum fluidization velocity and maximum pressure drop for gas–solid tapered fluidized beds. Experimental values of minimum fluidization velocity and maximum bed pressure drop with air as the fluidizing medium compare well with that predicted by the proposed models. The results have also been compared with other models available in the literature.

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1. Introduction

Most of the gas–solid fluidization behavior studies that have been reported are for straight cylindrical or columnar fluidized beds, although a considerable proportion of the fluidized beds have inclined walls or have a tapered bottom section. A velocity gradient exists in the axial direction, leading to unique hydrodynamic characteristics. Due to this characteristic, tapered fluidized beds have found wide applicability in many industrial processes such as, waste water treatment [1], immobilized biofilm reaction, incineration of waste materials, coating nuclear fuel particles, crystallization, coal gasification and liquefaction and roasting sulfide ores [2], food processing [3], etc.

Tapered fluidized beds are useful for fluidization of materials with a wide particle size distribution, as well as for exothermic reactions. They can be operated smoothly without any instability, i.e. with less pressure fluctuations [4] and also for extensive particle mixing [5,6]. In spite of its advantages and usefulness,

not much work has been reported in literature for understanding certain important characteristics, especially minimum fluidization velocity and maximum pressure drop. Studies have been reported by researchers to determine the factors affecting minimum fluidization velocity and maximum pressure drop. But some of these results are limited to regular particles only. Some of the previous investigations include fixed bed pressure drop calculations [7], flow regimes, incipient condition of fluidization, voidage distribution and bed expansion calculations [8] and development of a model for maximum pressure drop at incipient fluidization condition of a tapered fluidized bed [9]. The model developed by Shi et al. [9] is based on Ergun's equation and neglects friction between the particles and the wall.

Biswal et al. [10,11] developed theoretical models, for minimum fluidization velocity and pressure drop in a packed bed of spherical particles for gas–solid systems in conical vessels. Due to the angled walls, random and unrestricted particle movement occurs in a tapered bed with reduced back mixing [12].

Olazer et al. [13] compared their experimental results with that calculated using the models developed by Gelperin et al. [14] and Gorshtein and Mukhlenov [15] for maximum pressure

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Nomenclature

Ar	Archimedes No. = $gd_p^3(\rho_s - \rho_f)\rho_f/\mu_f^2$
C_1	$= (150(1 - \varepsilon_0)^2/\varepsilon_0^3)(\mu/(\phi_s d_p)^2)$
C_2	$= (1.75(1 - \varepsilon_0/\varepsilon_0^3))(\rho_f/\phi_s d_p)$
d_p	particle diameter (m)
D_0	bottom diameter of the tapered bed (m)
D_1	top diameter of the tapered bed (m)
Fr	Froude No. = $U_{mf}/\sqrt{gd_p}$
g	acceleration due to gravity (m s^{-2})
H_s	stagnant height of the particle bed (m)
ΔP_{\max}	maximum pressure drop through the particle bed (Pa)
ΔP_t	total pressure drop (Pa)
r_0	bottom radius of the tapered bed (m)
r_1	top radius of the tapered bed (m)
U_0	superficial velocity of the fluidizing fluid (m s^{-1})
U_{mf}	minimum fluidization velocity based on the bottom diameter of the tapered bed (m s^{-1})

Greek symbols

α	tapered angle ($^\circ$)
ε_0	voidage of the stagnant bed (–)
ϕ_s	sphericity of solid particle (–)
μ_f	fluid viscosity ($\text{kg m}^{-1} \text{s}^{-1}$)
ρ_f	fluid density (kg m^{-3})
ρ_s	solid density (kg m^{-3})

Subscript

exp.	experimental value
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drop and found that the predictions were not very accurate. They therefore proposed a modified equation for calculation of maximum pressure drop. Later, Peng and Fan [2] made an in-depth study of the hydrodynamic characteristics of solid–liquid fluidization in a tapered bed and derived theoretical models for the prediction of minimum fluidization velocity and maximum pressure drop, based on the dynamic balance of forces exerted on the particle. The experiments were however carried out for spherical particles only.

Jing et al. [16] and Shan et al. [17] developed models for gas–solid conical fluidized beds for spherical coarse and fine particles based on the Peng and Fan [2] models but neglected the pressure drop due to the kinetic change in the bed. Depypere et al. [3] have carried out studies in a tapered fluidized bed reactor and proposed empirical models for determination of expanded bed height by using static pressure and wall surface temperature measurements.

Therefore, it was felt necessary to develop a generalized correlation for the calculation of minimum fluidization velocity and maximum pressure drop in a tapered fluidized bed, which are the two important characteristics of fluidization for regular and irregular particles. In this study an empirical dimensionless correlation has been developed for predicting the minimum fluidization velocity and maximum pressure drop for regular and

irregular particles of gas–solid systems taking into account all the parameters, i.e., particle diameter, particle density, tapered angle, porosity and sphericity. The correlation for pressure drop however also depends upon the bed height. The applicability of the new model has been compared to that of existing models from literature.

2. Experimental details

2.1. Apparatus

A schematic diagram of the experimental set-up is shown in Fig. 1. The tapered columns were made of perspex sheets to allow visual observation with different tapered angles. The diameters of the column at the bottom were 48, 42, and 50 mm, where as at the top the diameters were 132, 174, and 212 mm, respectively. The column heights were 520, 504, and 483 mm, respectively. A 60 mesh screen at the bottom served as the support as well as the distributor. The calming section of the bed was filled with glass beads for uniform distribution of fluid. Two pressure taps, one just above the distributor and the other above the bed were provided to record the pressure drops. Pressure drop was measured by manometer, which was one meter long. Carbon tetrachloride (density = 1630 kg m^{-3}) was used as the manometric fluid. Air at a temperature of 310 K ($\rho_f = 1.17 \text{ kg m}^{-3}$ and $\mu_f = 1.8 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$), used as the fluidizing medium was passed through a receiver and a silica gel tower to dry and control the air flow before being sent through the tapered column. Two rotameters, one for the lower range (0–10 m^3/h) and the other for the higher range (10–120 m^3/h) were used to measure the air flow rates.

2.2. Procedure

The experiments were carried out in tapered columns having tapered angles of 4.61° , 7.47° and 9.52° . Different types of materials such as coal, sand, limestone, iron ore, refractory material, dolomite, sago (white colored spherical cereal) and glass beads were used for the investigation. All the experiments were carried out at a temperature of around 301 K and under one atmospheric pressure. The diameter of the spherical particles

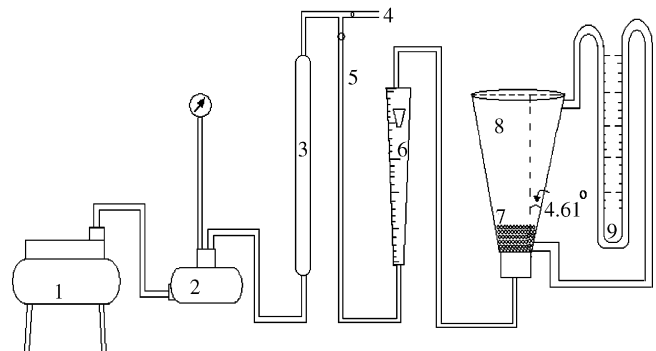


Fig. 1. Experimental set-up: (1) compressor, (2) receiver, (3) silica gel tower, (4) bypass valve, (5) line valve, (6) rotameter, (7) bed materials, (8) fluidizer, and (9) pressure tapping to manometer.

Table 1
Ranges of variables studied and characteristics of the bed

Material	Density (kg m ⁻³)	Voidage	Sphericity	Particle diameter (μm)	Tapered angle (°)	Static bed height (m)	U_{mf} (m s ⁻¹)	ΔP_{max} (Pa)
Coal	1545	0.46–0.465	0.65	717, 1200	4.61,9.52	0.092,0.107,0.13	0.19–0.34	755–2351
Sand	2638	0.42	0.72	717	9.52	0.092,0.107,0.13	0.26	1018–1506
Limestone	2785	0.245–0.305	0.85	500, 600, 800	4.61,7.47,9.52	0.092,0.107,0.13	0.17–0.59	850–2351
Sago	1303	0.35	1.0	1200	4.61	0.092,0.107,0.13	0.51	718–1018
Glass bead	2300	0.21–0.31	1.0	1000,2000, 3000	4.61,7.47,9.52	0.092,0.107,0.13	0.68–3.27	1134–2177
Dolomite	2785	0.36	0.79	717	9.52	0.092,0.107,0.13	0.32	1039–1566
Iron ore	5025	0.37–0.38	0.79	500, 600, 800	4.61	0.092,0.107,0.13	0.17–0.34	1320–1910
Refractory material	2610	0.44	0.69	717	9.52	0.092,0.107,0.13	0.26	1048–1495

(sago and glass bead) was determined by randomly selecting sample particles, measuring the diameters of individual particles by a vernier caliper, and calculating the average value. The diameter of the irregular particles (coal, sand, limestone, iron ore, refractory material and dolomite) was determined by sieve analysis. The density of the particles was obtained by dividing the weight of the particles by the volume of water displaced when the particles were placed in a cylindrical column filled with water.

The initial stagnant bed height was determined by taking a weighed amount of the material in the column and then passing water to fluidize the bed. The particles were then allowed to settle by gradually decreasing the flow rate till the valve was completely closed. The height of the bed was noted when there was no further change in the height. The experiment was repeated three times and the average height was taken as the initial stagnant bed height. For each of the experiment, after measuring the bed height, water in the column was drained out carefully till the surface of the bed was reached. Next, the water in the bed was drained out completely in to a separate container and the volume of water measured. The average volume of the water collected was taken as the void volume, from which the porosity of the stagnant bed was calculated. The sphericity was calculated using the equation:

$$\frac{(1 - \varepsilon_0)}{\phi_s} = 0.231 \log d_p + 1.417 \quad (1)$$

where d_p is the particle diameter in feet [18]. The procedure was repeated for all the particle sizes and materials used in each of the tapered columns. These parameters are summarized in Table 1.

To determine the minimum fluidization velocity and maximum pressure drop, the procedure followed by Lee et al. [19] was followed. The solid material was first charged to the column and air passed through it for about 5 min till the system was stable. The stagnant bed height was then recorded. The velocity of the air was increased incrementally allowing sufficient time to reach a steady state. The rotameter and manometer readings were noted for each increment in flow rate and the pressure drop and superficial velocity calculated. The velocity at which the pressure drop was maximum was taken as the minimum fluidization velocity. The same process was repeated for different

stagnant bed heights, different particles and different tapered angles of the tapered beds.

3. Model equations

Some of the well known correlations available for predicting the minimum fluidization velocity (U_{mf}) and maximum pressure drop (ΔP_{max}) for tapered beds are those by Peng and Fan [2] and Jing et al. [16]. Peng and Fan [2] developed a model for estimating minimum fluidization velocity and maximum pressure drop for solid–liquid system and spherical particles based on the dynamic balance of forces exerted on the particle. The correlation reported by them for minimum fluidization velocity is given in Eq. (2):

$$C_1 U_{mf} + C_2 \left(\frac{D_0}{D_1} \right) U_{mf}^2 - (1 - \varepsilon_0)(\rho_s - \rho_f)g \times \frac{(D_0^2 + D_0 D_1 + D_1^2)}{3D_0^2} = 0 \quad (2)$$

The equation for pressure drop has been developed from Ergun's equation, which also includes the pressure drop due to a kinetic energy change in the bed:

$$-\Delta P_{max} = C_1 H_s \frac{D_0}{D_1} U_0 + C_2 H_s \frac{D_0(D_0^2 + D_0 D_1 + D_1^2)}{3D_0^2} U_0^2 + \frac{1}{2} \left(\frac{U_0}{\varepsilon_0} \right)^2 \left[\left(\frac{D_0}{D_1} \right)^4 - 1 \right] \rho_f \quad (3)$$

Jing et al. [16] developed a model based on Ergun's equation for pressure drop calculation but neglecting the pressure drop due to the kinetic energy change in the bed. The equation proposed by them for nearly spherical particles is

$$-\Delta P_t = C_1 U_0 H_s \frac{r_0}{r_1} + C_2 U_0^2 H_s \frac{r_0(r_0^2 + r_0 r_1 + r_1^2)}{3r_1^2} \quad (4)$$

Based on the experimental data obtained in the present study, for different types of material, generalized correlations have been obtained by carrying out dimensionless analysis and estimating the constant coefficients by non-linear regression. The

Table 2
Comparison of experimental and calculated minimum fluidization velocity for stagnant bed height of 0.092 m

Materials	Tapered angle (°)	U_{mf} (m s ⁻¹)			Absolute error (%) based on experimental value	
		Experimental	Proposed model	Peng and Fan model	Proposed model	Peng and Fan model
Sago (1.2 mm)	4.61	0.51	0.44	0.52	13.73	1.96
Coal (1.2 mm)	4.61	0.34	0.29	0.29	14.71	14.71
Limestone						
0.5 mm	4.61	0.17	0.17	0.19	0.00	11.76
0.6 mm	4.61	0.20	0.21	0.25	5.00	25.00
0.8 mm	4.61	0.34	0.32	0.42	5.88	23.53
Glass beads						
1 mm	4.61	0.68	0.53	0.76	22.06	11.76
2 mm	4.61	1.34	1.24	1.51	7.46	12.69
3 mm	4.61	2.18	2.18	2.35	0.00	7.79
Iron ore						
0.5 mm	4.61	0.17	0.16	0.20	5.88	17.65
0.6 mm	4.61	0.20	0.21	0.24	5.00	20.00
0.8 mm	4.61	0.34	0.31	0.36	8.82	5.88
Limestone						
0.5 mm	7.47	0.20	0.23	0.26	15.00	30.00
0.6 mm	7.47	0.25	0.30	0.29	20.00	16.00
0.8 mm	7.47	0.42	0.45	0.47	7.14	11.90
Glass beads						
1 mm	7.47	0.85	0.73	0.94	14.11	10.59
2 mm	7.47	1.81	1.76	1.93	2.76	6.63
3 mm	7.47	3.02	3.11	3.54	2.98	17.22
Dolomite (0.717 mm)	9.52	0.32	0.35	0.29	9.38	9.38
Coal (0.717 mm)	9.52	0.19	0.22	0.16	15.79	15.79
Refractory material (0.717 mm)	9.52	0.26	0.28	0.22	7.69	15.38
Sand (0.717 mm)	9.52	0.26	0.30	0.23	15.38	11.54
Limestone						
0.5 mm	9.52	0.25	0.29	0.20	16.00	20.00
0.6 mm	9.52	0.42	0.36	0.36	14.29	14.29
0.8 mm	9.52	0.59	0.53	0.72	10.17	22.03
Glass beads						
1 mm	9.52	1.02	0.86	1.07	15.69	4.90
2 mm	9.52	2.18	2.08	2.68	4.59	22.94
3 mm	9.52	3.27	3.72	3.86	13.76	18.04

Table 3
Average absolute error (%) of maximum pressure drop by different models based on experimental values for all materials

Tapered angle (°)	Static height (m)	Average absolute error (%)		
		Proposed model	Peng and Fan model	Jing et al. model
4.61	0.092	7.15	22.30	22.28
7.47	0.092	7.05	18.38	17.21
9.52	0.092	12.64	20.05	19.21
4.61	0.107	6.12	21.30	21.03
7.47	0.107	6.38	21.21	20.52
9.52	0.107	11.63	25.13	24.46
4.61	0.130	8.17	21.31	20.81
7.47	0.130	4.81	22.31	21.52
9.52	0.130	17.39	27.51	26.95

dimensionless correlation for U_{mf} is given in Eq. (5) and the correlation for maximum pressure drop in Eq. (6):

$$Fr = 0.2714(Ar)^{0.3197}(\sin \alpha)^{0.6092} \left(\frac{\varepsilon_0}{\phi_s} \right)^{-0.6108} \quad (5)$$

$$\Delta P_{max} = 7.457 \left(\frac{D_1}{D_0} \right)^{0.038} \left(\frac{d_p}{D_0} \right)^{0.222} \left(\frac{H_s}{D_0} \right)^{0.642} \left(\frac{\rho_s}{\rho_f} \right)^{0.723} \quad (6)$$

4. Results and discussions

The minimum fluidization velocity and maximum pressure drop were calculated using Eqs. (5) and (6) respectively for regular and irregular particles in gas–solid system and compared with experimental data. The experimental data were also compared with the values calculated using Peng and Fan [2] as well as from the Jing et al. [16] model. The comparison of calculated and experimental minimum fluidization velocity for a stagnant bed height of 0.092 m, for different materials is shown in Table 2 and the maximum pressure drop in Table 3 (taking percent absolute error for all materials and calculating the average absolute error). The minimum and maximum absolute error for maximum pressure drop calculation by proposed model for each material based on experimental values are as follows: sago (2.35% and 10.56%), glass bead (0.25% and 27.16%), coal (7.06% and 15.69%), dolomite (18.72% and 19.07%), refractory material (9.44% and 12.67%), sand (11.19% and 16.81%), limestone (1.91% and 17.72%) and iron ore (1.01% and 11.68%).

Fig. 2 shows the experimental total pressure drop as a function of superficial gas velocity for a tapered angle of 9.52° and a bed height of 0.13 meter for dolomite. From point A to point B, the total pressure drop increases with the increase of superficial gas velocity. The transition from the stagnant bed to partially

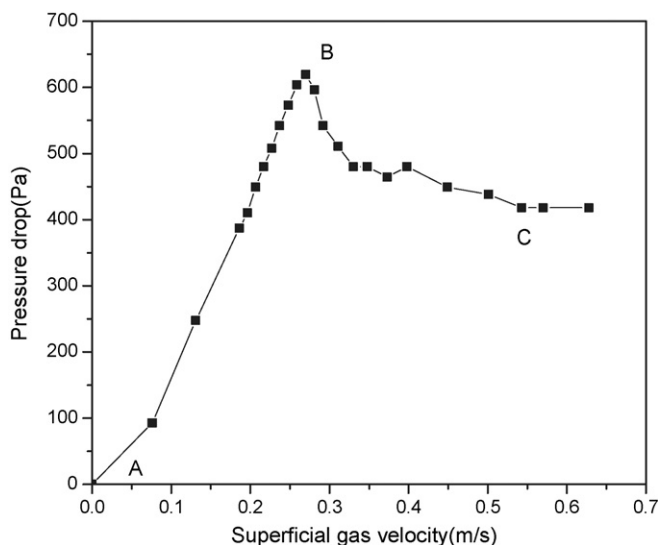


Fig. 2. Effect of superficial gas velocity on pressure drop ($\alpha = 9.52^\circ$ and $H_s = 0.13$ m, particle = dolomite).

fluidized bed occurs at point B, i.e. at the minimum fluidization velocity and maximum total pressure drop. From point B, the total pressure drop decreases with the increase of superficial gas velocity and from point C it remains constant. From point B to point C, the bed is partially fluidized and thereafter it is fully fluidized. Correlations developed in the present study can be used for estimating the pressure drop and fluidization velocity at point B.

The comparison of the calculated values of U_{mf} from Eqs. (2) and (5) with experimental values (as given in Table 2) is presented in Fig. 3a–c, for a stagnant bed height of 0.092 m. Substituting the calculated values of U_{mf} into the Eqs. (3), (4) and (6), the predicted values of maximum pressure drop through the tapered bed, ΔP_{max} , can be obtained. The comparison of the predicted and measured pressure drop is illustrated in Fig. 4a–c for different tapered angles. Good agreement was obtained between the predicted and measured values. The errors in the case of tapered angle of 4.61° and 7.47° are small where as in the case of tapered angles of 9.52° are slightly larger. A slightly larger error arises due to the extent of variation of superficial gas velocity along the axial direction, which is more pronounced in the tapered bed with a larger tapered angle. This can lead to decrease in the predictive capability of Eqs. (5) and (6). It can also be seen that the prediction by the proposed model is better than that by Peng and Fan [2] and Jing et al. [16] model, for gas–solid systems.

According to Eq. (2), D_1 increases with increasing cone angle, which causes the minimum fluidization velocity and maximum pressure drop to also increase. In other words, the operation range for the stagnant bed regime becomes wider by increasing the tapered angle. The proposed model for U_{mf} has been verified against experimental data given by Depypere et al. [3], for a gas–solid system. For sucrose–starch beads, the experimental U_{mf} reported by them is 0.085 m s^{-1} whereas that calculated by using the proposed model is 0.084 m s^{-1} (taking $\varepsilon = 0.445$, $\phi_s = 1.0$, $\rho_f = 1.09 \text{ kg m}^{-3}$, $\mu_f = 0.000019 \text{ kg m}^{-1} \text{ s}^{-1}$, ρ_f and μ_f are at 50°C). The error was within 1%, which proves that it could be applicable to other gas–solid systems showing its unified nature.

4.1. Effect of tapered angle

Fig. 5 shows the effect of tapered angle of the bed on flow regimes, where bed-tapered angles are 4.61° , 7.47° and 9.52° and the stagnant bed height is 0.092 m. It can be seen from the figure that the maximum pressure drop, ΔP_{max} , and the minimum fluidization velocity, U_{mf} , increase with the increase in tapered angles. This has also been reported by Peng and Fan [2] and Jing et al. [16].

4.2. Effect of stagnant bed height

It was experimentally observed that the U_{mf} was not a function of stagnant bed height in conical tapered beds. This phenomenon was also observed by Povrenovic et al. [20] and Caicedo et al. [21].

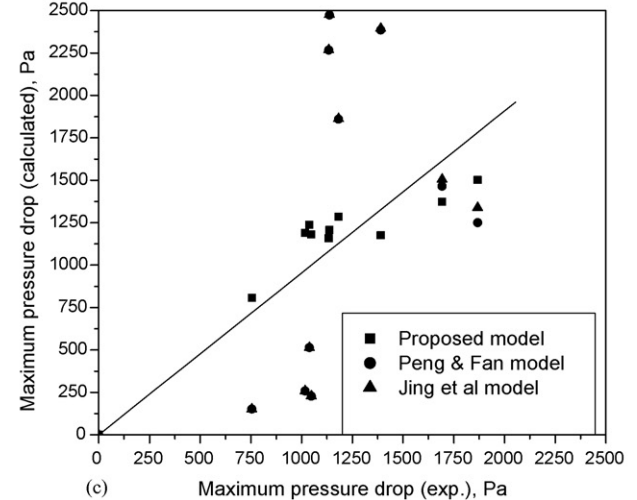
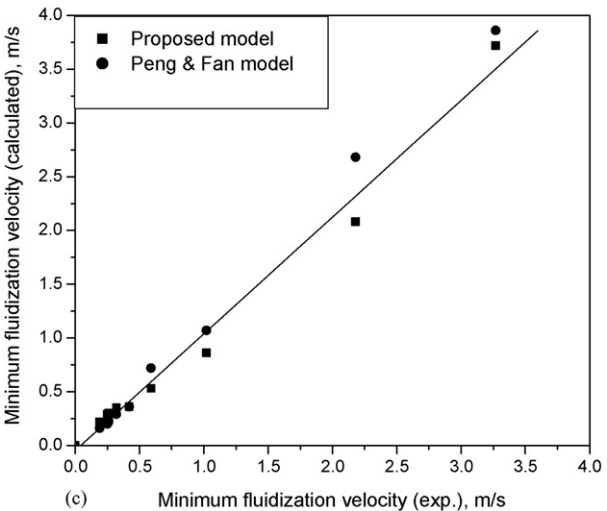
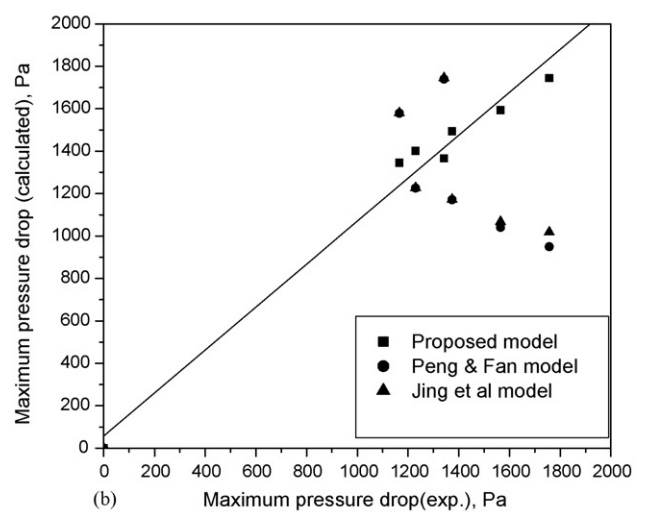
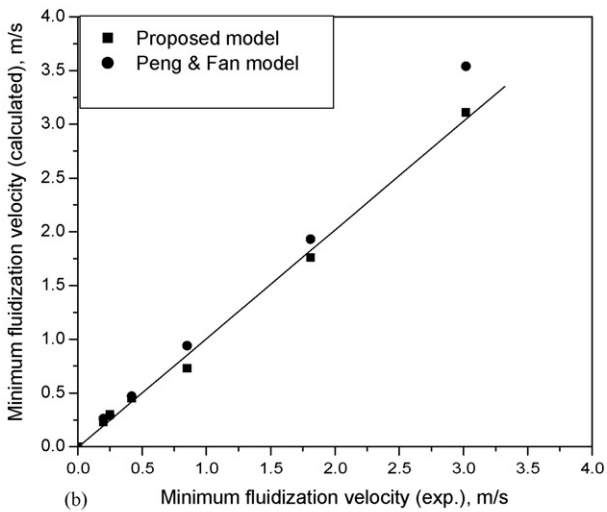
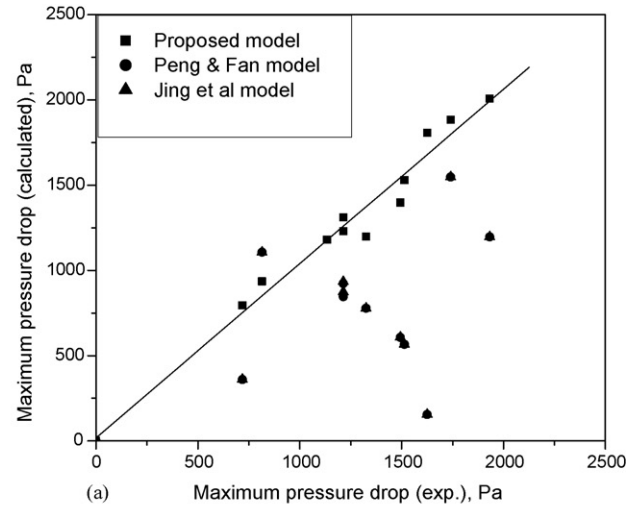
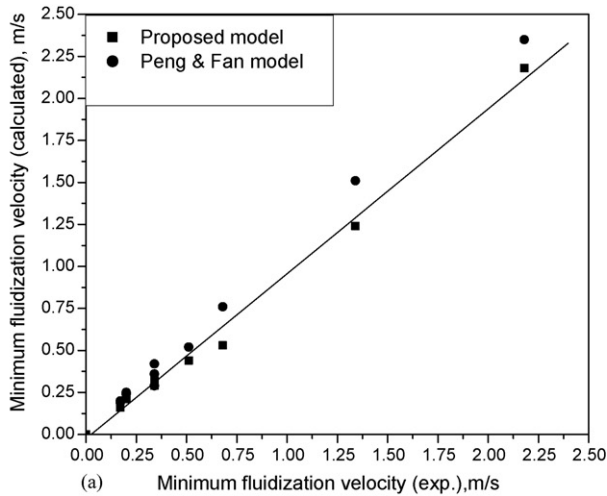


Fig. 3. (a) Comparison of calculated minimum fluidization velocity with experimental values ($H_s = 0.092\text{ m}$, $\alpha = 4.61^\circ$, particles = sago, coal, limestone, glass bead and iron ore), (b) comparison of calculated minimum fluidization velocity with experimental values ($H_s = 0.092\text{ m}$, $\alpha = 7.47^\circ$, particles = limestone and glass bead), and (c) comparison of calculated minimum fluidization velocity with experimental values ($H_s = 0.092\text{ m}$, $\alpha = 9.52^\circ$, particles = dolomite, coal, refractory-material, sand, limestone and glass bead).

Fig. 4. (a) Comparison of calculated maximum pressure drop with experimental values ($H_s = 0.092\text{ m}$, $\alpha = 4.61^\circ$, particles = sago, coal, limestone, glass bead and iron ore), (b) comparison of calculated maximum pressure drop with experimental values ($H_s = 0.092\text{ m}$, $\alpha = 7.47^\circ$, particles = limestone and glass bead), and (c) comparison of calculated maximum pressure drop with experimental values ($H_s = 0.092\text{ m}$, $\alpha = 9.52^\circ$, particles = dolomite, coal, refractory-material, sand, limestone and glass bead).

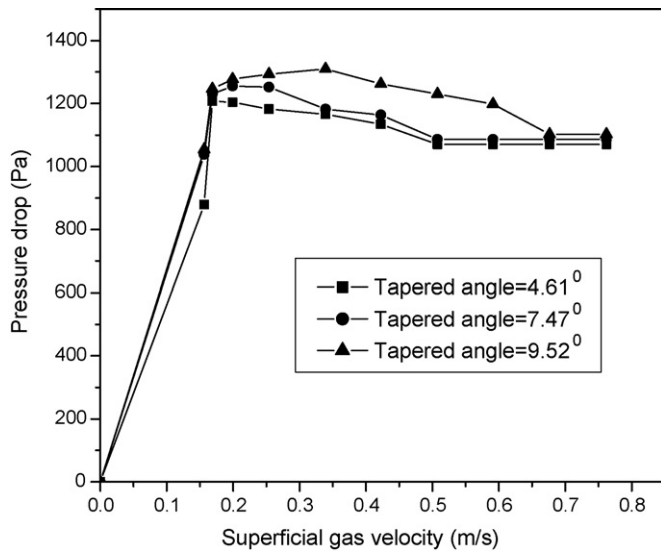


Fig. 5. Effect of tapered angle on fluidization ($H_s = 0.092$ m, particle = limestone).

5. Conclusion

Experiments were carried out with a number of gas–solid systems in a tapered column to study certain important characteristics of the fluidized bed. Generalized empirical correlations, based on dimensionless analysis, have been developed for predicting the minimum fluidization velocity and maximum pressure drop for these systems. The constant coefficients for these correlations were obtained by non-linear regression analysis. The deviation of the calculated minimum fluidization velocity and maximum pressure drop, using Peng and Fan [2] model and Jing et al. [16] model, from the experimental values were quite high. The model was also tested with data reported by other authors and was found to predict with an error of less than 1%. It was also found that the correlation for the calculation of minimum fluidization velocity (proposed model and Peng and Fan model) was very sensitive to porosity and hence care should be taken to measure the porosity very accurately. However, in the case of pressure drop correlation, the effect of porosity was insignificant (in the range of porosity studied) and hence has not been included in the correlation. The error in predicting the pressure drop was higher compared to that for predicting the minimum fluidization velocity. Further work need to be carried out to improve upon the models. However, since studies on tapered fluidized bed are limited, the present study would provide some insight into the behaviour of different solid–gas systems. The proposed correlations could also find

practical utility in designing and operation of tapered fluidized beds for various gas–solid systems.

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